

Session 2: I Can See For Tiles And Tiles

Opener

1. Check out this floor from a church in Seville, Spain:



Taken from <http://en.wikipedia.org/wiki/File:Semi-regular-floor-3464.JPG>

Recreate this pattern using your cut-out regular polygons. If you extended your pattern in every direction forever, what would be the ratio of the number of triangles to squares to hexagons that you'd need?

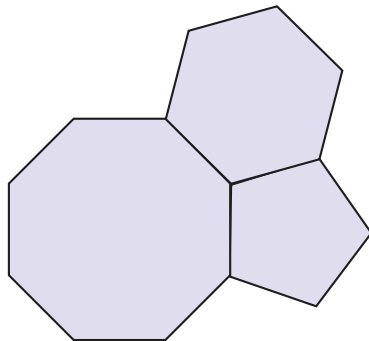
Maybe a barber named Figure-o laid out these tiles.

. . . come to think of it, why was the Barber of Seville speaking Italian the whole time?

Hopefully you didn't toss your tiles!

Important Stuff

2. Last time, Ashley put a regular pentagon, hexagon and octagon together so they share a vertex. Does this work out? Explain why or why not.



Girl look at that poly.
Girl look at that poly.
Girl look at that poly.
It works out! (maybe)

I'm hexy and I know it.

3. Look at all the arrangements of regular polygons that you made on Set 1. For each arrangement, calculate $V - E + F$, where V is the number of vertices, E is the number of edges, and F is the number of polygon faces in the arrangement.

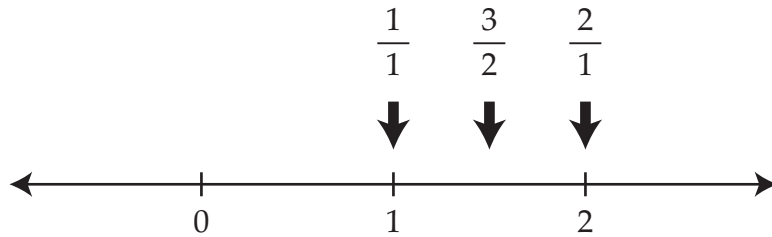
Include interior edges and vertices. $V - E + F$ is pronounced "vuhf".

4. Start with 0 and 1, then keep adding two terms to get the next:

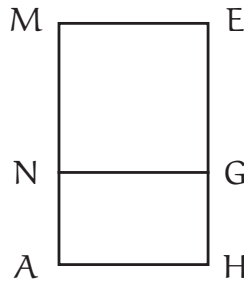
$$0, 1, 1, 2, 3, 5, 8, 13, \dots$$

Eventually these numbers start multiplying like rabbits. Wait, no, they're adding like rabbits? EH.

Use consecutive Fibonacci numbers (value/previous value) to make fractions and place these fractions on this number line. Keep placing! What do you notice?



5. You can take any non-square rectangle and chop a square out of it! In the diagram below, MEGN is a square chopped out of rectangle AHEM.



AHEM! May I have your attention please! Thanks, carry on.

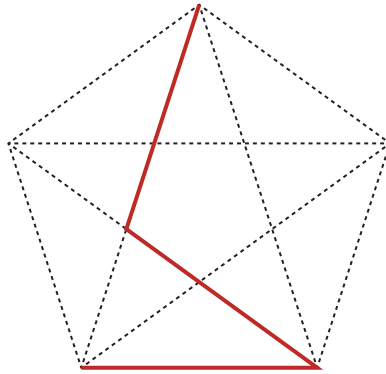
- a. Suppose $EH = 2$ and $AH = 1$. Is rectangle GHAN a scaled copy of the original rectangle?
- b. Suppose $EH = 3$ and $AH = 2$. Is GHAN a scaled copy of AHEM this time?
- c. What if $EH = 5$ and $AH = 3$? Well, shoot.
- d. If $EH = x$ and $AH = 1$, write a proportion that would have to be true if GHAN is a scaled copy of AHEM.

GHAN in 60 Seconds?
GHAN but not forgotten.

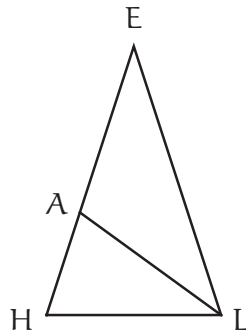
GHAAAANNNNNNNNNNNNN!

What's x ? EH.

- e. If $AH = 1$, estimate the length of EH to two decimal places.
6. Leon measured these three marked segments in a regular star pentagram. He couldn't decide which was longest. Which one is longest? Explain how you know.



7. Here's the center triangle from Problem 6.



Last time, this was Triangle 2. Now, it's got a triangle, too!

- a. Which triangle is a scaled copy of which other triangle? Be as precise as you can.
- b. Give a brief explanation of *why* these triangles are scaled copies.
- c. Replace these question marks in a way that must form a true proportion:

$$\frac{EH}{??} = \frac{??}{??}$$

- d. If $LH = 1$, estimate the length of EH to two decimal places.

Two objects are *similar* if they are scaled copies. Congruent objects are similar, with scale factor 1.

Wait, which length is that? EH .

Neat Stuff

8. The golden ratio ϕ can be defined several ways. One way is that it's a positive number so that $\phi^2 = \phi + 1$.

This letter is pronounced like either "fee" or "fie".

- a. Using the definition above, without using or figuring out the value of ϕ , show that $\phi^3 = \phi(\phi + 1)$.
 - b. Show that $\phi^3 = \text{blah}\phi + \text{bleh}$. You figure out the blahnks, but they're integers.
 - c. Show that $\phi^4 = \text{blih}\phi + \text{blöh}$, again without evaluating ϕ .
 - d. Show that $\phi^5 = \text{bluh}\phi + \text{blyh}$.
 - e. Describe a general rule for ϕ^n . Awesome!!
9. There are some ways to fit 3 regular polygons together perfectly to surround a vertex. Find as many as you can and write them as ordered triples (a, b, c) where $a \leq b \leq c$ are the number of sides in the three polygons.
10. For each triple you found in Problem 9, there's something interesting about the sum of the numbers, the product, or the sum of the reciprocals. One of those.
11. Use what you found in Problem 10 to make a complete list of *all* the ways that 3 regular polygons can fit together at a vertex.
12. About what proportion of that Seville floor's area is made up of hexagons? squares? triangles?
13. Here's an interesting fact about ϕ^{-10} :

$$\phi^{-10} = \text{EH} \cdot \phi + \text{AH}$$

where EH and AH are integers. Find EH and AH by yourself, then verify the result with technology. What does this tell you about the value of ϕ ?

Tough Stuff

14. A regular pentagon has side length 1. How far is it from one vertex to the midpoint of the opposite side?
15. a. Find the three regular polygons that come the closest to fitting together at a vertex, but don't because they overlap.
- b. Find the three regular polygons that come the closest to fitting together at a vertex, but don't because they leave a super-tiny gap.

ϕ^3 is pronounced "foe".

ϕ^4 is pronounced "fum". That smell is either the blood of an Englishman, or Gambino's Pizza.

Sheesh, could they be any vaguer here? EH.

How's it going, EH?

I don't think that pizza is in Kansas anymore, Toto.